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Thermal erosion of magnetoplasmadynamic thruster cathode

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INTRODUCTION

The magnetoplasmadynamic (MPD) arcjet finds many practical applications in the areas of electric space propulsion, plasma spray and metal cutting torches, etc. In the spacecraft, the MPD arc thrusters operate at large discharge electric current and low mass flow rates in order to obtain high specific impulse and thrust efficiency. For these space applications, a life time of 10^7 cycles (~ 100 days) would be demanded for the space MPD thruster [1]. In this condition, the MPD thruster would be exposed to severe thermal environments, thus, resulting in a sharp temperature rise at the cathode root due to the pulsed heat inputs which largely affect electrode erosion and electron emission. It is required for satisfactory and reliable operation of the thruster, that the cathode be operated at a high temperature at the cathode tip to allow the emission of electrons with a moderate electric field, so as to suffer minimum material loss. But severe surface thermal erosion at the cathode tip has been noticed experimentally in transient as well as steady state operating conditions.

Shih *et al.* [2] have not considered the conical shape of the cathode in their analysis of heat conduction problem with radiation and also its effect on the temperature profile. In practice, a 2% thoriated tungsten cathode whose $L/2r$ ratio varies from 1.5 to 3.0 with a semi cone angle of 15–30° (see in Fig. 1) is commonly used in cascade and MPD arc devices. Thermal phenomena on the electrode surface have been analyzed numerically and experimentally by Kuwahara *et al.* [3]. They solved numerically, the unsteady heat conduction equation using finite difference method under periodic heat input condition over a flat plate. The conical shape cathode has been analysed previously using Runge–Kutta method [4] and finite difference scheme [5], without considering the thermal erosion of the cathode material.

The objective of this paper is to present a numerical solution of the heat transfer problem of conical shape cathode. A deforming element technique [6] is applied to take into account ablation of the cathode material. The solution is advanced by fully explicit time-stepping in conjunction with the finite element method.

PROBLEM FORMULATION

Consider a cathode with a varying cross-section area and with constant thermophysical properties. The configuration of a MPD thruster is illustrated in Fig. 1. The energy equation

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NOMENCLATURE

a	cross-sectional area of cathode	R	electrical resistivity
$\mathbf{C}$	heat capacitance matrix	r	cathode radius
$\mathbf{C}_L$	lumped capacitance matrix	T	local temperature of cathode at x
c	heat capacity of the material	T_a	ablation temperature
$\mathbf{g}$	thermal load vector	T_c	coolant temperature
h	heat transfer coefficient	T_∞	surrounding temperature
I	applied electric current	t	time
$\mathbf{K}$	conductivity matrix	Δt	computing time
$\mathbf{K}_d$	convective matrix	x	space coordinate.
k	thermal conductivity		
L	cathode length	Greek symbols	
L_f	latent heat of the material	α	semi cone angle
l	element length	ε	emissivity
N	finite element shape function	ρ	material density
P	perimeter of cathode	κ	thermal diffusivity
q	heat flux at cathode root	σ	Stefan–Boltzmann constant.

with combined conduction, convection, radiation and Ohmic heating can be written as

$$\rho c A(x)\frac{\partial T}{\partial t} = k\frac{\partial}{\partial x}\left[A(x)\frac{\partial T}{\partial x}\right] + hP(x)(T-T_\infty) + \varepsilon\sigma A(x)(T^4 - T_\infty^4) + I^2 RA(x) \quad (1)$$

with the boundary conditions

$$T = T_c \quad \text{at} \quad x = 0, \quad t > 0 \quad (2a)$$

$$-k\frac{\partial T}{\partial x} = q \quad \text{at} \quad x = L, \quad t > 0 \quad \text{and} \quad T < T_a \quad (2b)$$

$$-k\frac{\partial T}{\partial x} = q - \rho L_f \frac{\mathrm{d}x}{\mathrm{d}t} \quad \text{at} \quad x = L, \quad t > 0 \quad \text{and} \quad T > T_a \quad (2c)$$

and initial condition

$$T = T_i \quad \text{at} \quad t = 0 \quad \text{for all}\ x. \quad (2d)$$

DEFORMING FINITE ELEMENT FORMULATION

It is appropriate to discretize the heat conduction equation in terms of deforming finite elements in order to treat ablation at the surface. The spatial shape function N_j in the Galerkin weak form of the equations will be time-dependent

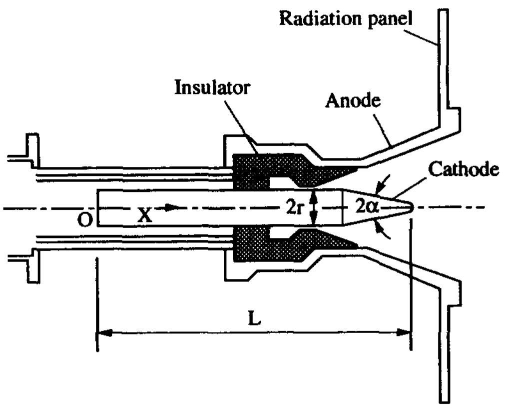

Fig. 1. MPD thruster.

at every fixed location x_j. The time derivative of the temperature is discretized into the time rate of change of the coordinate and shape function, N. Using the isoparametric element and the Langrangian nature of the finite element approximation [6], the time derivative can be written as

$$\frac{\partial T}{\partial t} = N_j\frac{\mathrm{d}T_j}{\mathrm{d}t} - u_j \nabla T, \quad (3)$$

where u_j denotes the local deformation velocity at x_j. The last term in the equation (3) can be viewed as a transport term to account for the convective effect of grid motion. Following the usual Galerkin procedure, one obtains the heat conduction equation in the form of a system of semidiscretized differential equation in terms of nodal temperature, collected in the global vector $\mathbf{T}$ as

$$\mathbf{C}\dot{\mathbf{T}} + (\mathbf{K} - \mathbf{K}_d)\mathbf{T} = \mathbf{g}, \quad (4)$$

where superscript dot denotes time partial differentiation, and $\mathbf{C}$, $\mathbf{K}$, $\mathbf{K}_d$, and $\mathbf{g}$ are the heat capacitance matrix, the conventional thermal conductivity matrix, the convective matrix accounting for mesh deformation due to erosion and the thermal local vector, respectively. On any element e, with nodes i and j, the element matrices can be written as

$$\mathbf{C}^e = \frac{\rho c l^e}{12}\begin{bmatrix} 3A_i + A_j & A_i + A_j \\ A_i + A_j & A_i + 3A_j \end{bmatrix}$$

$$\mathbf{K}^e = \frac{k}{2l^e}(A_i + A_j)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{K}_d^e = \frac{\rho c}{12} \times \begin{bmatrix} -\{A_i(3u_i+u_j)+A_j(u_i+u_j)\} & \{A_i(3u_i+u_j)+A_j(u_i+u_j)\} \\ -\{A_i(u_i+u_j)+A_j(u_i+3u_j)\} & \{A_i(u_i+u_j)+A_j(u_i+3u_j)\} \end{bmatrix}$$

$$\mathbf{g}^e = f^e + \frac{hl^e}{12}\begin{bmatrix} 3P_i + P_j & P_i + P_j \\ P_i + P_j & P_i + 3P_j \end{bmatrix} - \frac{(h+h^*)l^e T_\infty}{6}\begin{bmatrix} 2P_i + P_j \\ P_i + 2P_j \end{bmatrix} + \frac{I^2 R l^e}{6}\begin{bmatrix} 2A_i + A_j \\ A_i + 2A_j \end{bmatrix},$$

where

$$h^* = \varepsilon\sigma(T_\infty^2 + T^2)(T_\infty + T)$$

and f^e is for boundary condition at the cathode root.

Using the lumped capacitance matrix and fully explicit time stepping [7], equation (4) is rewritten at time $(t+\Delta t)$ in the form

$$\sum_{I=1}^{m} (\mathbf{C}_L)_I \Delta T = -\Delta t[(\mathbf{K}-\mathbf{K}_d)-\mathbf{g}]_I, \quad (5)$$

where ΔT_I denotes the increment in temperature at nodes I over the time step Δt.

Before advancing the solution, the allowable stability time step Δt for each element according to

$$\Delta t \leqslant \frac{(l^e)^2}{2\kappa_e}$$

is computed, where κ_e is the element diffusivity and l^e is a representative element length. Due to the erosion of the cathode conical tip, the characteristic length of the element will change as the time proceeds.

RESULTS AND DISCUSSION

A FORTRAN computer program has been written for the calculation of the temperature distribution along the cathode and the erosion rate. The computations have been made on a CDC CYBER 170/730 digital computer. The following thermophysical properties of tungsten were taken in the numerical analysis:

$$k = 163.7\ \mathrm{W\ (mK)^{-1}}, \quad c = 134.0\ \mathrm{J\ (kgK)^{-1}},$$

$$\rho = 19350.0\ \mathrm{kg\ m^{-3}}\ [8], \quad \varepsilon = 0.4\ [8], \quad h = 100\ \mathrm{W\ m^{-2}}\ [4],$$

$$R = 5.5\times10^{-8}\ \Omega\ \mathrm{m}, [9].$$

To validate the algorithm, the computations were performed for a cathode length of 2.54 cm, a radius of 0.3 cm, a semicone angle of 15° and applied current strength of 1000 A. There are 20 elements and time step of 0.5 ms used in the heat transfer calculations. The temperature at the cathode root is taken as 3000 K which is below the ablation temperature, and the other end of the cathode is maintained at the coolant temperature of 300 K. The transient and steady temperature profiles along the cathode are calculated and compared with the numerical results of refs. [4] and [5] and are found to be in good agreement. These analyses are reported in ref. [11].

After validating the finite element program, the numerical analysis is carried out to obtain the erosion rate due to high heat flux at the cathode root. The value of heat flux at the cathode root is taken as 1.2 GW $\mathrm{m^{-2}}$ [3]. The geometrical parameters considered are $L = 2.0$ cm, $r = 0.3$ cm and $\alpha = 30°$. The applied electric current strength of 1500 and 1750 A are used in heat transfer analysis. The values of ablation temperature and latent heat of fusion are taken from ref. [10] as 3700 K and 183.754 kJ $\mathrm{kg^{-1}}$, respectively. Before attaining the ablation temperature, equation (5) is solved subject to boundary conditions (2a) and (2b), and neglecting the convective matrix, $\mathbf{K}_d$. As soon as the cathode root reaches the ablation temperature, i.e. $T \geqslant T_a$, the boundary condition (2b) is changed to (2c). The rate of material loss and local deformation velocity are calculated and used to compute the convective matrix $\mathbf{K}_d$. Equation (5) is now solved to get the temperature distribution along the cathode. Figure 2 depicts the temperature profiles along the cathode after 20.0 s. It can be seen from the figure that the temperature decreases rapidly along the nose cone portion of the cathode, while in the remaining length of the cathode $(0 \leqslant x \leqslant 1.0\ \mathrm{cm})$ the temperature varies almost linearly. Higher temperature in the conical portion of the cathode is found for $I = 1750$ A as compared to $I = 1500$ A. It is observed that the Ohmic heating produces considerable amount of heat generation in the conical region of the cathode. The erosion rate is obtained as 3.156 μg m $\mathrm{C^{-1}}$ for $I = 1750$ A and $q = 1.2$ GW $\mathrm{m^{-2}}$. The erosion rate is compared with the experimental data of Kuriki *et al.* [1] and found to be in agreement. The present analysis can be extended easily under pulse operating condition of the MPD arcjet thruster.

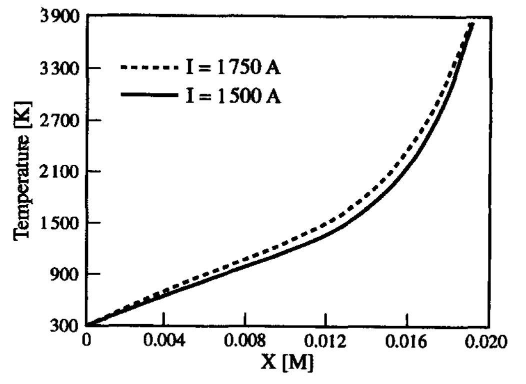

Fig. 2. Temperature distribution along the cathode.

CONCLUSIONS

Heat transfer analysis of a conical shaped cathode has been studied using a finite element method. The thermal erosion is considered using a deforming element formulation. The temperature and erosion rate is found to be higher for higher electric current strength. The Ohmic heating is significant in the conical portion of the cathode. The erosion rate of 3.156 μg m $\mathrm{C^{-1}}$ is found using present numerical analysis and is in agreement with result of Kuriki *et al.* [1].

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